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Analytical Approximations

Volume 4

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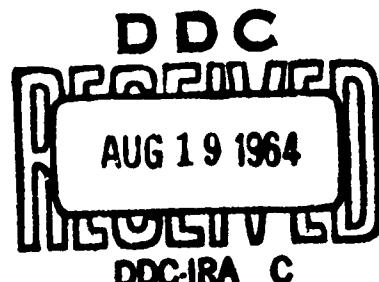
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11-3-52

### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .00037 over  $(0, \infty)$ ,

$$q(.5, .5+y) \doteq 1 - \frac{.1045}{[1 + .129y + .079y^2 + .056y^3]^4}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the  $q(R, R+y)$  surface for any  $R > 0$  and for  $y$  ranging over  $(0, \infty)$ .

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0007 over  $(0, \infty)$ ,

$$q(1, 1+y) \doteq 1 - \frac{.267}{[1 + .203y + .079y^2 + .062y^3]^4}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the  $q(R, R+y)$  surface for any  $R \geq 0$  and for  $y$  ranging over  $(0, \infty)$ .

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0011 over  $(0, \infty)$ ,

$$q(4, 4+y) \doteq 1 - \frac{.45}{[1 + .227y + .064y^2 + .065y^3]^4}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the  $q(R, R+y)$  surface for any  $R > 0$  and for  $y$  ranging over  $(0, \infty)$ .

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(r^2+x^2)} I_0(r x) r dr$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0013 over  $(0, \infty)$ ,

$$\begin{aligned} \lim_{R \rightarrow \infty} q(R, R+y) &= \int_{-\infty}^y \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt \\ &\doteq 1 - \frac{.5}{[1 + .209y + .061y^2 + .062y^3]^4} \end{aligned}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the  $q(R, R+y)$  surface for any  $R \geq 0$  and for  $y$  ranging over  $(0, \infty)$ .

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .006 over  $(0, \infty)$ ,

$$\lim_{R \rightarrow 0} \frac{1 - q(R, R+y)}{1 - q(R, R)} = e^{-\frac{1}{2}y^2}$$
$$= \frac{1}{[1 + .015y + .076y^2 + .040y^3]^4}.$$

The above gives information concerning a degenerate limiting case in the approximating of fixed-R semi-cross-sections of the  $q(R, R+y)$  surface for  $y$  ranging over  $(0, \infty)$ .

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